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A novel strategy of pareto-optimal solution searching in multi-objective particle swarm optimization (MOPSO)☆

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ABSTRACT

In multi-objective particle swarm optimization (MOPSO) algorithms, finding the global optimal particle (*gBest*) for each particle of the swarm from a set of non-dominated solutions is very difficult yet an important problem for attaining convergence and diversity of solutions. First, a new Pareto-optimal solution searching algorithm for finding the *gBest* in MOPSO is introduced in this paper, which can compromise global and local searching based on the process of evolution. The algorithm is implemented and is compared with another algorithm which uses the Sigma method for finding *gBest* on a set of well-designed test functions. Finally, the multi-objective optimal regulation of cascade reservoirs is successfully solved by the proposed algorithm.

1. Introduction

During the past decades, heuristic multi-objective optimization algorithms have been thoroughly investigated mainly because of the fact that they can be suitably applied to find multiple Pareto-optimal solutions in one single simulation run [1,2]. By applying these algorithms to different optimization problems, researchers have demonstrated that algorithms are more pragmatic and efficient compared with classical approaches [3].

Particle swarm optimization (PSO) [4,5] is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behaviors of bird flocking or fish schooling. PSO has been extensively applied in various optimization problems due to its unique searching mechanism, excellent convergence and simple implementation. PSO is particularly suitable for multi-objective optimization mainly because of the high speed convergence that the algorithm presents for single-objective optimization [6–9]. In recent years, various studies have been published on multi-objective particle swarm optimization (MOPSO) in different fashion [6,7,10–12]. In MOPSO, the global optimal solutions are a set of non-dominated solutions. Furthermore, the conceptual barrier of *gBest* and *pBest* tends to get blurred in the multi-objective application of the basic PSO. Consequently, although it is an important problem for attaining convergence and diversity of solutions, choosing *gBest* and *pBest* from the set of Pareto-optimal solutions for each particle of the swarm to direct its flight is still very difficult [10].

In this paper, we propose a new algorithm for choosing *gBest* for each particle of the swarm from a Pareto-optimal solutions set. The implementation results show that by using the proposed algorithm in a MOPSO, we can achieve a good convergence and diversity of solutions. Also, we solve the multi-objective optimal regulation of cascade reservoirs by successfully adopting the MOPSO algorithm, and obtain the non-inferior solutions set of the problem of Three Gorges cascade multi-objective regulation. This paper has the following structure: the new Pareto-optimal solution searching algorithm is

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presented in Section II. In section III, the structure of MOPSO is described. The experiments are explained and the problem of multi-objective regulation of cascade reservoirs based on MOPSO is solved in Section IV. Finally, conclusions are obtained in Section 5.

2. PARETO-optimal solution searching algorithm

It is very important in MOPSO to select the best global particle (*gBest*) from the non-dominated solutions set for each particle of the swarm to attain convergence and diversity solutions, which we call the process of Pareto-optimal solution searching [10]. In single-objective PSO, the *gBest* is determined easily by selecting the particle which has the best position from the swarm, while in multi-objective optimization problems, the optimum solutions are a set of Pareto-optimal solutions and since each particle in the swarm should fix one of the Pareto-optimal solutions as its *gBest*, the *gBest* of each particle may be different. Problems in Pareto-optimal solution searching lie in: (I) it is difficult to define criteria to select *gBest* because of the particles in non-dominated solutions set do not dominate each other and none or very little information can be obtained on the basis of the partial order defined by the dominance relation; (II) Pareto-optimal solution searching should compromise the ability of global and local searching to improve the diversity of solutions and convergence. (III) Pareto-optimal solution searching should not be restricted by the dimension of objective space.

The Dominated tree method [7] and the Sigma method [10] are the representative existing Pareto-optimal solution searching algorithms. The Dominated tree method stores the elite particle and facilitates the choice of *gBest* for each particle in the swarm, but this method does not utilize density information; the Sigma method can guide particles to the Pareto-optimal font directly, and find solutions with good convergence. If the initial value is null or bad-distributed of Archive set, it may cause premature in certain case (e.g., in multifrontal problems). So, the global searching ability of the method remains to be improved.

A new Pareto-optimal solution search algorithm is presented in this section. First, we estimate the density values of the particles in non-dominated solutions set. The algorithm generates grids to divide the search objective space explored so far, and locates the particles in non-dominated solutions set by using these grids as a coordinate system where each particle's coordinate is defined according to the value of its objective function. The density value of each particle is the number of members of the grid that this particle is situated in. The greater the number of particles in the grid, the greater is the density value of the particle, and in otherwise turns to less. The particles, which have a smaller density value in the non-dominated solutions set, are set higher selection pressures to explore wider search objective space. In addition, the Sigma method is employed to improve the local searching ability. Then, in the process of evolution, we assess the ability of global and local searching on line by calculating the number of non-dominated solutions found so far. If it has a larger number of non-dominated solutions in the search process, the Sigma method can be used with greater probability to improve the search accuracy and convergence speed, and otherwise, the global search ability of the algorithm can be enhanced by selecting particles having smaller density values with a greater probability in the non-dominated solutions set. Based on that, when the particle A_j in non-dominated solutions set is selected as the *gBest* of the particle P_i in the population, the fitness of selection intensity is calculated from the following equation:

$$Fit(i,j) = \lambda_t \cdot \frac{\max(\Delta\sigma_i)}{\Delta\sigma_{i,j}} + (1-\lambda_t) \cdot \frac{\max(g_j)}{g_j} \tag{2.1}$$

where, $\lambda_t = \frac{n_t}{|A|}$ stands for the ability of global or local searching, $|A|$ denotes the fixed size of the non-dominated solutions set A, n_t denotes the number of members in the non-dominated solutions set at t-th generation, g_j is the number of members of the grid that A_j is situated in, $\max(g_j) = \max_{j\in|A|}\{g_j\}$, $\Delta\sigma_{i,j} = |Sigma(P_i) - Sigma(A_j)| + \varepsilon$ denotes the distance of Sigma value of P_i and A_j, ε is a small positive number, $\max(\Delta\sigma_i) = \max_{j\in|A|}\{\Delta\sigma_{i,j}\}$, for 2-dimensional optimization problem, $Sigma(P_i)$ is calculated with [10]:

$$Sigma(P_i) = \frac{(K_2 f_1)^2 - (K_1 f_2)^2}{(K_2 f_1)^2 + (K_1 f_2)^2} \tag{2.2}$$

where, f_1 and f_2 are objective values of P_i, K_1 and K_2 are the maximum values of the first and second objective values of P_i.

The particle which has the maximal fitness value is selected as the *gBest* of P_i. The Pareto-optimal solution search algorithm of P_i is as follows:

Step 1: Compute $\Delta\sigma_{i,j}$ and $\max(\Delta\sigma_{i,j})$ of P_i

A. $\sigma_i = Sigma(P_i)$;

B. FOR $j = 1$ TO $|A_t|$

$\sigma_j = Sigma(A_j)$;

$\Delta\sigma_{i,j} = distance(\sigma_i, \sigma_j)$;

End;

C. $\max(\Delta\sigma_{i,j}) = \max\{\Delta\sigma_{i,j}, j = 1, 2, \ldots, |A_t|\}$;

Step 2: Estimate the density values of the particles in A_t
A. $g_j = Density(A_j), j = 1, 2, \ldots, |A_t|$;
B. $\max(g_j) = \max\{g_j, j = 1, 2, \ldots, |A_t|\}$;
Step 3: Select *gBest* of P_i
A. Compute $Fit(i, j)$ by Eq. (2.1), $j = 1, 2, \ldots, |A_t|$;
B. $k = \{j| \max\{Fit(i, j), j = 1, 2, \ldots, |A_t|\}\}$

where, A_t is the non-dominated solutions set at t-th generation, $distance(\sigma_i, \sigma_j)$ computes the distance of σ_i and σ_j, $Density(A_j)$ computes the density values of the j-th particles in non-dominated solutions set A_t, k denotes the serial number of *gBest* of P_i.

3. Description of MOPSO algorithm

As the analogy of the proposed algorithm with the classic multi-objective evolutionary algorithms (MOEAs), a secondary population, the so-called Archive set, is maintained, which contains a representation of the non-dominated front among all solutions considered so far. The Archive set may just be used as external storage and must be updated at each generation. The structure of bi-group MOPSO is shown as follows:

Step 1: $(P_1, A_1) = Initialization$
Step 2: FOR $t = 1$ to N
A. $P_{t+1} = Generate(P_t, A_t)$
FOR $j = 1$ TO *POPSIZE*
$g_{j,t} = FindgBest(A_t, P_{j,t})$
$P_{j,t+1} = UpdateParticle(P_{j,t}, g_{j,t})$
$Evaluate(P_{j,t+1})$
$p_{j,t} = UpdatepBest(P_{j,t+1})$
NEXT
B. $A_{t+1} = UpdateArchive(P_{t+1}, A_t)$
C. $P_{t+1} = Mutation(P_{t+1})$
NEXT
Step 3: $OutputArchive(A_{t+1})$

where, t denotes the generation index, P_t is the population, A_t is the Archive set at t-th generation, $g_{j,t}$ is the *gBest* of j-th particle, $p_{j,t}$ is the *pBest* of j-th particle, and $P_{j,t}$ is the j-th particle of P_t at t-th generation. The function *Initialization* generates the initial population and copies all non-dominated solutions to the Archive set, the function *Generate*, generates the next generation population, *FindgBest* selects *gBest* from A_t for $P_{j,t}$ adopting the Pareto-optimal solution search algorithm, *UpdateParticle* updates the speed and position of $P_{j,t}$ using $g_{j,t}$ and $p_{j,t}$, *Evaluate* evaluates the particles of population, *UpdateArchive* inserts the non-dominated solutions of P_{t+1} to A_t and removes the superfluous particles from A_t, and *OutputArchive* outputs the particles of the Archive set. The steps of the MOPSO algorithm are iteratively repeated until the maximum number of generations is reached.

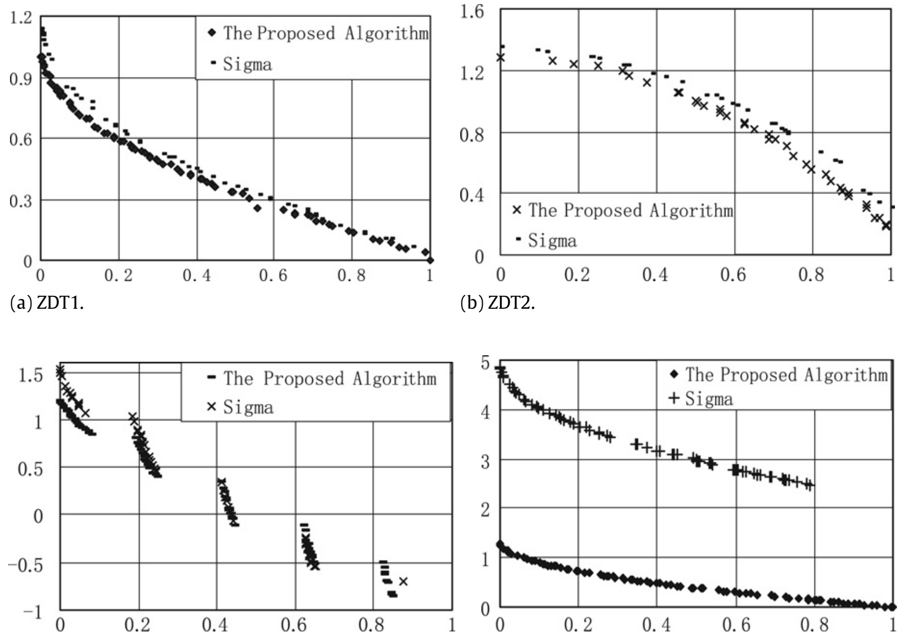
4. Experiments and application

In this part, the proposed algorithm is compared against the Sigma method based on some test functions to show its potential competence. Two algorithms have same structures which are shown in Section III, the latter uses the Sigma method for finding *gBest* of each particle in the swarm from the Archive set. Then, the Three Gorges cascade multi-objective optimal regulation problem is solved by employed the proposed algorithm to illustrate its efficiency and engineering practicability.

4.1. Experimental results and comparison based on test functions

Reference [3] suggested a systematic way to develop test problems for multi-objective optimization, and constructed four test problems in which four are chosen here, called the ZDT1, ZDT2, ZDT3 and ZDT4. The ZDT1, ZDT2 and ZDT3 have convex, nonconvex and discontinuous Pareto-optimal front respectively. The problem ZDT4 has 21^9 different local Pareto-optimal fronts in the search space, but only one corresponds to the global front. The Euclidean distance in the decision space between solutions of two consecutive local Pareto-optimal sets is 0.25, therefore not all local Pareto-optimal sets are distinguishable in the objective space.

In all the following examples, 2000 generations are carried out, population size is chosen $N = 100$, the size Archive set is set to 100, the number of decision variables of the test problems is equal to 30, the learning rates c_1, c_2 are 2.05, and the inertia weight w is taken from 0.9 to 0.4 with a linear decreasing rate. For each test problem, 30 independent runs are executed.



(a) ZDT1. (b) ZDT2. (c) ZDT3. (d) ZDT4.

Fig. 1. Comparison of the proposed method and Sigma method applied to ZDT1(a), ZDT2(b), ZDT3(c) and ZDT4(d).

Table 1
C metrics of the proposed algorithm in comparison with Sigma method.

Test functions	ZDT1	ZDT2	ZDT3	ZDT4
$C(A, B)$	0.7	0.94	0.41	1
$C(B, A)$	0.11	0	0.1	0

We present results of using the proposed algorithm, then we compare this algorithm with the Sigma method which has same structure as Section III and function *FindgBest* is implemented as introduced by Ref. [10]. Fig. 1, form (a) to (d), shows the graphical results produced by two algorithms on the test problems ZDT1, ZDT2, ZDT3 and ZDT4. As is shown in these figures, the proposed algorithm has a better diversity and convergence than the Sigma method. The convergence of solutions can be tested by using the C metric [3]. Table 1 shows the comparative results considering the C metrics, where A represents the proposed algorithm and B represents Sigma method. For the ZDT2 and ZDT4, C (Sigma method, the proposed algorithm) are equal to 0 and C (the proposed algorithm, Sigma method) are equal to 0.94 and 1, and it means that none of the solutions of the Sigma method can weakly dominate the solutions of the proposed algorithm.

4.2. Optimal regulation of cascade reservoir based on MOPSO

In the uncertainty electricity market environment, the issue of cascade optimal regulation is a large scale, dynamic, nonconvex and nonlinear multi-objective optimal problem, which is under constraint conditions of electrical market trading rules, hydrological cycle, generation control, power system security and reliability, power demand and consumers' reaction. It is difficult to handle the problem by traditional regulation methods. In this section, the Three Gorges cascade optimal regulation problem with two objective functions which including maximum power generation benefit and maximum firm power and considering monthly design flow of Three Gorges basin as reservoir inflow, is solved by adopting the proposed algorithm.

Problem formulation

Objective function

Power generation benefit and firm power which are non-commensurable and inversely proportional [13], are two key index of cascade reservoir regulation. The objective function with power generation benefit and firm power of cascade hydropower station are given by the following:

$$\max \sum_{i=1}^{N} \sum_{t=1}^{T} N_t \cdot \Delta T \tag{4.1}$$

$$\max[\min(N_t), t = 1, 2, \ldots, T] \tag{4.2}$$

where, N is number of hydropower stations, T is number of periods, N_t is output power of i-th hydropower station at t-th time period. Eq. (4.1) is used to maximize the value of annual power generation and Eq. (4.2) is used to firm power of the cascade hydropower station.

Constraints

(1) Hydropower station models

The generator net MW output is calculated as a non-linear function of head level and turbine discharge, the output power of the hydropower station at t-th period is as follows

$$N_t = \sum_{j=1}^{M} f(Q_j, h_j) \tag{4.3}$$

where, Q_j is discharge of j-th generator in t-th period, h_j is head level of hydropower station, and M is number of generators in the hydropower station.

(2) Water reservoir balance

Water reservoir level at the end of a period depends on the water reservoir level at the beginning of the period, inflow during the period, and discharge for generation in the period and the spillage in the period. In terms of energy equivalent, water reservoir balance can be written as

$$W_t = W_{t-1} + I_t - G_t(N_t) - SP_t \tag{4.4}$$

where, W_t stands for water reservoir level in t-th period, I_t is expected inflow, G_t is generating flow which is non-linear function of output power N_t, and SP_t is spillage in t-th period.

(3) Generation upper and lower limit

$$N_{min} \le N_t \le N_{max}. \tag{4.5}$$

(4) Reservoir upper and lower limit

$$W_{min} \le W_t \le W_{max}. \tag{4.6}$$

Handling constraints

It is very difficult to solve multi-objective optimal regulation of cascade reservoirs due to complex constraints. Based on the characteristics of a cascade hydropower system, we change the constraints to the feasible region of the water level in cascade reservoirs, and then the evolution of the particles of the swarm is restricted to this region. Thereby, the constrained optimization problem is changed to unconstrained optimization problems. The algorithm for calculating the feasible region of particles at t-th generation is shown as follows:

Step 1: calculating the discharge lower limit of hydropower station in upper reaches with the limit of the discharge in down reaches:

$$\underline{Q}'_t = V_D(Z_{D,t+\tau}) - V_D(Z_D) + (\underline{Q}_D - Q_q)\Delta T; \qquad \underline{Q}_t = \max(\underline{Q}_t, \underline{Q}'_t).$$

Step 2: calculating water level limit $\overline{Z}_1$ and $\underline{Z}_1$ corresponding to Z_{t-1}:

$$\overline{Z}_1 = Z(V(Z_{t-1}) + (I_t - \underline{Q}_t)\Delta T); \qquad \underline{Z}_1 = Z(V(Z_{t-1}) + (I_t - \overline{Q}_t)\Delta T).$$

Step 3: calculating water level limit $\overline{Z}_2$ and $\underline{Z}_2$ corresponding to Z_{t+1}:

$$\overline{Z}_2 = Z(V(Z_{t+1}) - (I_{t+1} - \overline{Q}_{t+1})\Delta T); \qquad \underline{Z}_2 = Z(V(Z_{t+1}) - (I_{t+1} - \underline{Q}_{t+1})\Delta T).$$

Step 4: calculating water level limit $\overline{Z}_3$ and $\underline{Z}_3$ with the generation upper and lower limit $\overline{N}_t$ and $\underline{N}_t$ adopting trial-and-error method.

Step 5: assuming that the water level upper and lower limits are $\underline{Z}_0$ and $\overline{Z}_0$ at t-th period.

Step 6: calculating water level limit at t-th period:

$$\overline{Z}_t = \min(\overline{Z}_0, \overline{Z}_1, \overline{Z}_2, \overline{Z}_3); \qquad \underline{Z}_t = \max(\underline{Z}_0, \underline{Z}_1, \underline{Z}_2, \underline{Z}_3)$$

where, $Z_t, I_t, \overline{Z}_t$ and $\underline{Z}_t$ denote water level, reservoir runoff, upper limit and lower limit of water level at t-th period respectively, $\underline{Z}_D$, $\underline{Q}_D$ and Q_q stand for the lower limit of the water level, the lower limit of discharge and inter-zone inflow of the hydropower station in down reaches respectively, τ is water arrival time of cascade hydropower stations, $V()$ is the function that calculates reservoir capacity and $Z()$ calculates the reservoir water level based on the reservoir water level-capacity curve.

The results of regulation

Design flows of which the probabilities are 0.01 and 0.99 are taken as inflow, 5000 generations are carried out, the population size is chosen as 100, the size Archive set is set to 100, the number of decision variables is equal to 72 (the number of periods and hydropower stations are 36 and 2 respectively). The regulation results are shown in Fig. 2, where the x-coordinate denotes annual power generation (10^8 kWh) and y-coordinate firm power (10^8 kW) of hydropower stations.

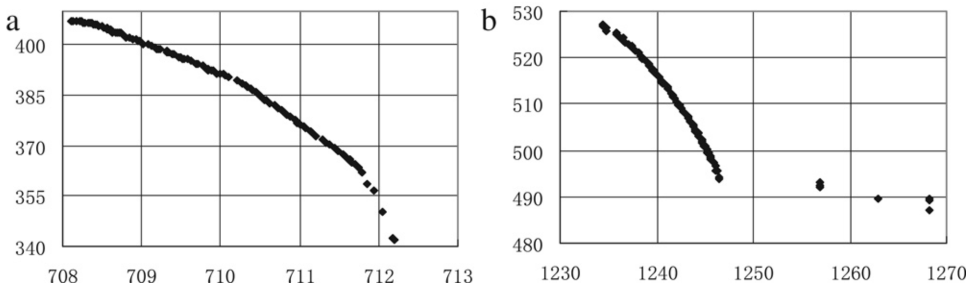


Fig. 2. Results of multi-objective optimal regulation in cascade reservoirs of Three Gorges.

Fig. 2(a) illustrates that the annual power generation varies from 407.0 to 712.2 and firm power from 407.0 to 342.1, and the inverse relation between annual power generation and firm power of hydropower stations is significant at low flow years.

If the regulating ability of the reservoir is enough to regulate the inflow completely in a high normal flow regulation period, the firm power will not be reduced with increasing annual power generation. However, the Three Gorges Reservoir with seasonal regulation cannot regulate the inflow completely during the year. Consequently, the regulation of the reservoir may cause a water discharge loss in flood periods but insufficient output in low inflow periods, and Fig. 2(b) demonstrates this case that the variation range of annual power generation is 1065.6–1058.8 and the firm power is 471.3–468.0.

5. Conclusions

In this paper, we propose a new Pareto-optimal solution searching algorithm in MOPSO for finding the *gBest* of the particles in a swam from a non-dominated solutions set in order to quickly converge towards a Pareto-optimal front of high diversity, this algorithm can compromise global and local searching based on the process of evolution. Experiments have shown that this algorithm, which can find solutions with good diversity and convergence, is an efficient approach for solving complex multi-objective optimization problems. Through solving the Three Gorges cascade optimal regulation problem and obtaining the non-dominated solution set of problems concerning power generation benefit and firm power, the regulation results can offer scientific warranty for operation of the Three Gorges cascade reservoir and show the effectiveness of the presented algorithm with regard to solving large scale complex multi-objective optimization problems.

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